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Feasibility study on missile launch detection and trajectory tracking

Wong, Chee Mun Kelvin

Monterey, California: Naval Postgraduate School

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NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

THESIS

FEASIBILITY STUDY ON MISSILE LAUNCH DETECTION AND TRAJECTORY TRACKING

by

Chee Mun Kelvin Wong

September 2016

Thesis Advisor: Oleg A. Yakimenko
Second Reader: Fotis A. Papoulias

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This thesis explores the challenges in creating a simulation program to process video footage from an unstable platform and the limitations of using background subtraction method to detect the missile motion. Although the simulation program test results showed that it is unable to consistently detect a missile launch and track its trajectory for all the test videos; the developed algorithms allowed a surveillance UAV to detect a missile launch for most of the videos and also track its trajectory with an accuracy that is sufficient for targeting purposes.

This thesis is limited to using the simulation program to detect a launch event offline and is based on the amateur rocket launch data gathered during the launch trials at Mojave Desert in May of 2016.

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FEASIBILITY STUDY ON MISSILE LAUNCH DETECTION AND TRAJECTORY TRACKING

Chee Mun Kelvin Wong
Civilian, Defence Science Technology Agency
B.Eng., Nanyang Technological University, 2005

Submitted in partial fulfillment of the
requirements for the degree of

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NAVAL POSTGRADUATE SCHOOL
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Approved by: Professor Oleg A. Yakimenko
Thesis Advisor

Professor Fotis A. Papoulias
Second Reader

Garth V. Hobson
Chair, Department of Mechanical and Aerospace Engineering

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LIST OF ACRONYMS AND ABBREVIATIONS

CUAV	Counter Unmanned Aerial Vehicle
ICBM	Intercontinental Ballistic Missile
IDF	Israel Defense Forces
ORB	Oriented FAST and Rotated BRIEF
RGB	Red-Blue-Green
SIFT	Scale-Invariant Feature Transform
SURF	Speeded-Up Robust Feature
UAV	Unmanned Aerial Vehicle

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I. INTRODUCTION

A. BACKGROUND

Missiles and rockets have been deployed in many military operations throughout the years. They have been used as offensive and defensive weapons against munitions, aircraft, buildings, vehicles, and many other high-value targets in the theatre. In July 2014, Hamas attacked Israel by firing a series of rockets on neighboring cities [1], [2]. In response, the Israel Defense Force (IDF) deployed the Iron Dome air defense system to counter the rockets that were approaching these populated areas [3].

Missile detection and tracking, using either active or passive sensors mounted on a UAV, is essential in effecting any missile defense. Active sensors such as radar utilize a transmitter to bounce a radio frequency off the target and a receiver to capture the signal reflected from the target. These active sensors, though, are detectable by the anti-radar sensors deployed by the adversary. In response, the adversary can identify the tracking system's location and launch countermeasures against it.

An UAV utilizing passive sensors, such as a thermal-imaging camera or a color-imaging camera, have the advantage of detecting and tracking the target without alerting an adversary. The passive sensor does not transmit any energy toward the target; instead, it utilizes video imagery from its sensor and identifies the target from the video footage. Therefore, the susceptibility of the UAV against countermeasures during operations is reduced significantly.

In order to track and detect the missile passively using the UAV's passive sensor; the target needs to have a distinctive signature in the video footage. The UAV can detect an adversary's missile by processing the thermal imagery from the thermal-imaging sensor, which captures the temperature gradient of the surroundings within its field of view. As the missile's propulsion motor emits gases at high temperature to generate the thrust required for its flight, the heat

signature of the propulsion motor will be detected by the thermal-imaging sensor. However, in order to achieve a high temperature gradient between the missile's propulsion motor and the surroundings, the thermal-imaging sensor needs to be cooled down before the start of every operation. This increases the setup time for the passive sensor on the UAV, and the coolant mechanism increases the UAV's overall weight.

On the other hand, the UAV can use a color-imaging sensor to detect and track the adversary's missile. As the color-imaging sensor captures video footage of everything within its field of view, the UAV has to identify the missile from the rest of the surroundings before it can detect its launch and track its trajectory. However, the passive sensor does not require any additional hardware for operation and has a very short setup time.

This thesis addresses the problem of processing a missile launch video, taken from a color-imaging sensor, on a simulation program to detect a missile launch and track its trajectory. As the initial missile launch videos, taken from a quadcopter, are not stabilized, it is not suitable for processing by the simulation program. Therefore, additional missile launch videos, taken from a stabilized platform on the ground, are recorded in the missile launch trial held in Mojave Desert on May 21. These missile launch videos are then used to write the simulation program and to assess the program's performance. Once the simulation is able to detect a missile launch and track its trajectory, it will be able to calculate the missile speed and position. This information can be relayed to an attacking UAV to launch a countermeasure against it. As the simulation program is unable to process any missile launch videos taken from an unstable platform, it cannot be deployed to a surveillance UAV to detect a missile launch and trajectory tracking in real time. If the simulation program can be enhanced to stabilize the video before performing any video processing, it can be deployed to a surveillance UAV for real-time missile launch detection and trajectory tracking. The next section describes the current approaches to address this problem and the final section of this chapter provides a detailed layout of this thesis.

B. PREVIOUS WORK ON MOTION TRACKING AND DETECTION

There are several computer vision methods used to detect and track an object from video footage. Methods such as background subtraction, Scale-Invariant Feature Transform (SIFT) and Speeded-Up Robust Feature (SURF) detection, and Kalman filtering are frequently used for object tracking. These methods have been applied frequently on video records of sportsman in their sports event for the coaches to find ways to improve the performance of their sportsman.

In motion detection and tracking, it is always desired to have the motion of the object in interest, left in the frame after applying the background subtraction method on successive frames. However, due to the camera motion during filming, the surroundings will also move according to the camera motion. This will cause the surroundings to appear as noise in the frame after applying the background subtraction method. Therefore, the video image has to be stabilized before applying background subtraction, to reduce the noise created by the surroundings movement, so that the object motion can be detected easily by the simulation program. Lucas and Kanade presented a method to identify features in two images that could be used as an anchor to stabilize the video footage [4]. Szeliski presented a method [5] that can automatically register video frames into 2D and partial 3D scene models. This method allows common features in successive images to be identified and stitched together to form a video mosaic.

With the video image stabilized, the targeted object in the video frame could be identified using descriptors of the unique features of the object. This could be done using the SIFT method [6] and the SURF method [7]. Rublee et al. presented an alternative method to SIFT and SURF, which improves on their performance and efficiency of feature detection [8].

Once the object had been identified in the video image, the motion of the object could be estimated through the comparison of the object's location in successive video frames. Patel used successive frame differencing and pixel

variance to track, detect, and validate a moving object in a video sequence [9]. Fleet and Weiss described a variety of mathematical models that could be used to determine the object's motion [10].

C. THESIS ORGANIZATION

This thesis consists of four chapters. Chapter I provides a brief introduction to the purpose of missile detection and tracking and an overview of previous work on the topic of motion tracking and detection. It also formulates a problem this thesis tries to address.

Chapter II gives an overview of the method used in a simulation program to detect missile launch and track its trajectory. A detailed breakdown of the methods used and the lessons learnt are discussed in this chapter.

Chapter III discusses the test results of the simulation program on several missile launch videos. The performance of the simulation program is presented in this chapter.

Chapter IV discusses the methods to perform pose estimation of the missile and compares the estimates from the simulation program with an actual firing.

Chapter V discusses the feasibility of using the simulation program to detect a missile launch and tracks its trajectory. This chapter also recommends future work that could expand on this study.

The Python code for the simulation program and the summarized results of all the tested missile launch videos are given in Appendix A and Appendix B, respectively.

II. ALGORITHMS FOR MISSILE LAUNCH DETECTION AND TRAJECTORY TRACKING

This chapter addresses the video stabilization issues encountered in processing the unstable video footage taken from a quadcopter in previous rocket launch and describes the new experimental setup that is used to capture the rocket launch videos. From the rocket launch videos, the algorithms of the simulation program are written based on the characteristics of the missile launches during the trial. Subsequently, the chapter will describe the essence of the simulation program written in Python using open-sourced libraries from Opencv that are commonly used for video processing.

The simulation program is split into two main sections, namely a missile launch detection phase and a missile trajectory tracking phase. The simulation program first breaks down the videos into individual frames and converts them to grayscale. The frames have fewer variables to process in grayscale than in Red-Green-Blue (RGB) color format. Once the frames have been processed for each phase, the results are back projected to the frames in RGB format and saved as color video footage. The following sections address the problem of video stabilization and describe the simulation program algorithm.

A. VIDEO STABILIZATION

Two rocket launch videos (each taken from a different perspective by a quadcopter) were initially used to write the simulation program. Sample frames are shown in Figure 1. Due to the stability of the quadcopter during flight, the video footage of the rocket launches was not very stable. This posed a great challenge in detecting the rocket launch and tracking its trajectory using the video processing tools available.

Figure 1. Images of Missile Launch from Two Different Angles

In the most common video processing for motion, the background subtraction method is used to sieve out any objects that are moving in the video. It subtracts the intensity of two video frames and creates a new frame, in grayscale, to show the objects that were moving between the frames. As shown in Figure 2, the background subtraction method detects the rocket launch and also the surrounding features due to the motion of the quadcopter during the recording. The video processing tools in Opencv cannot stabilize the video footage enough for the simulation program to perform its task.

Figure 2. Images of Missile Launch after Background Subtraction

Therefore, we stabilize the videos using external video processing software, and the results are shown in Figure 3.

Figure 3. Background Subtraction Images after Video Stabilization

The results of the background subtraction images after video stabilization are good for launch detection and trajectory tracking. However, as the motion of the video footage is too pronounced, the stabilization results are not consistent in all the video frames. Some of the frames still have the surrounding features captured by the background subtraction method. This causes the simulation program to falsely detect a missile launch and also mistake a patch of grass as a missile in flight. Therefore, the two missile launch videos taken from a quadcopter are not suitable for use by the simulation program.

To simplify the problem of detecting a missile launch and tracking its trajectory, another series of missile launch videos is taken from a camera mounted onto a tripod on the ground so that there are no issues of video stabilization.

B. EXPERIMENTAL SETUP

The missile launch trials are held in the Mojave Desert on May 21, 2016. A total of six missile launch videos is recorded using a Canon 650D during the trial. The camera is capable of recording videos in 1080p (1920 x 1080 pixels) with a frame rate of 25 frames per second. The range of the camera to the missile launch site is not measured during the trial as the main objective is to record missile launch videos to be used as test videos for creating the simulation program. The experimental setup is shown in Figure 4.

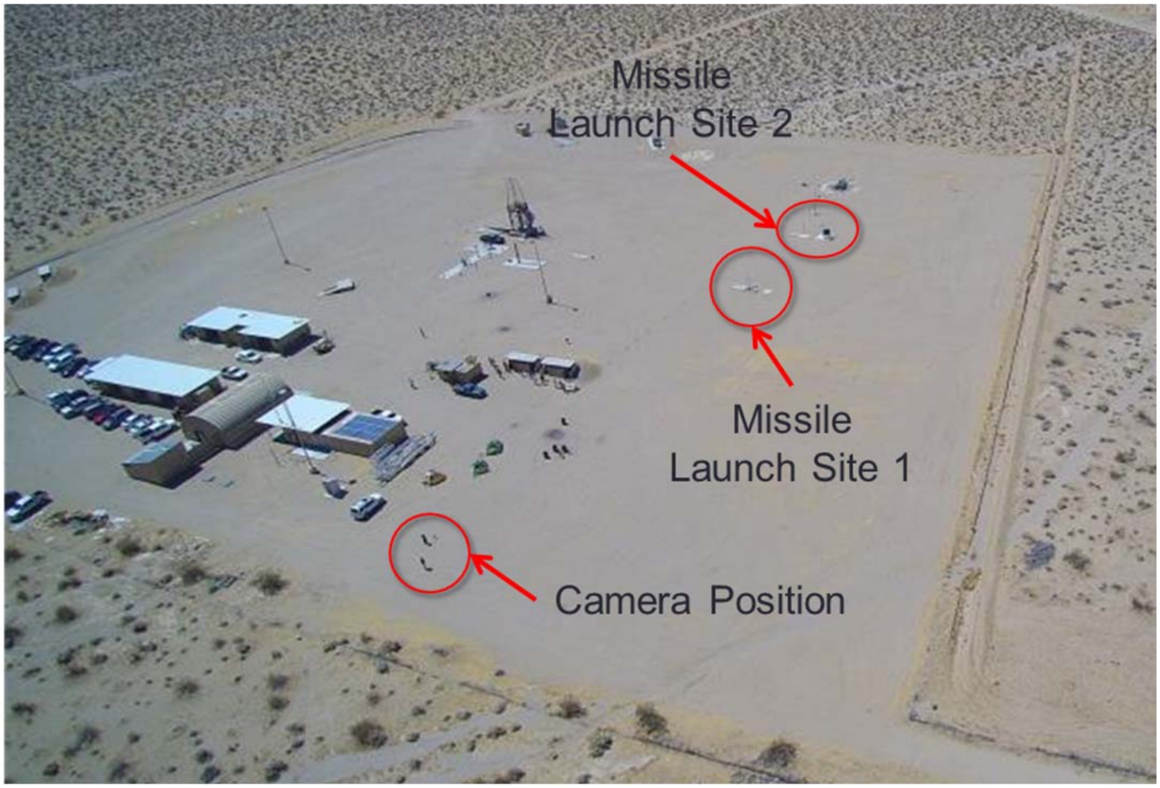

Figure 4. Experimental Setup of the Missile Launch

C. SIMULATION PROGRAM ALGORITHM

As mentioned earlier, the simulation program has two main phases, namely the missile launch detection phase and the missile trajectory tracking phase. When a missile launch has been detected in the first phase, the program transitions to the missile trajectory tracking phase and attempts to track the missile's trajectory. In this phase, the program first locates the missile in each frame and tracks its position as the missile moves across the camera's field of view. The simulation program marks the missile location with a red circle on each of the video frames to indicate a successful track and outputs it to the missile video. An overview of the simulation program algorithm is shown in Figure 5.

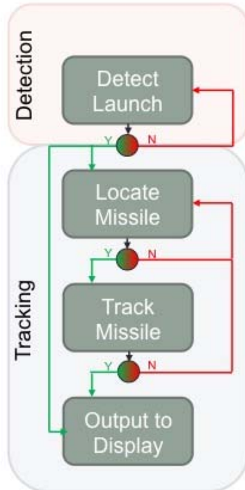

Figure 5. Overview of the Simulation Program Algorithm

1. Missile Launch Detection Phase

When the missile is launched from the launch pad, smoke plumes are generated by the rocket motor to provide the thrust needed to propel the missile. Therefore, the smoke plume is a good indication that a missile has been launched. Through the detection of smoke plumes of a certain size in each frame, the simulation program can identify a missile launch. As different missiles produce different sized smoke plumes during launch (Figure 6), the size of the smoke plume that can be used for missile launch declaration, have to be optimized.

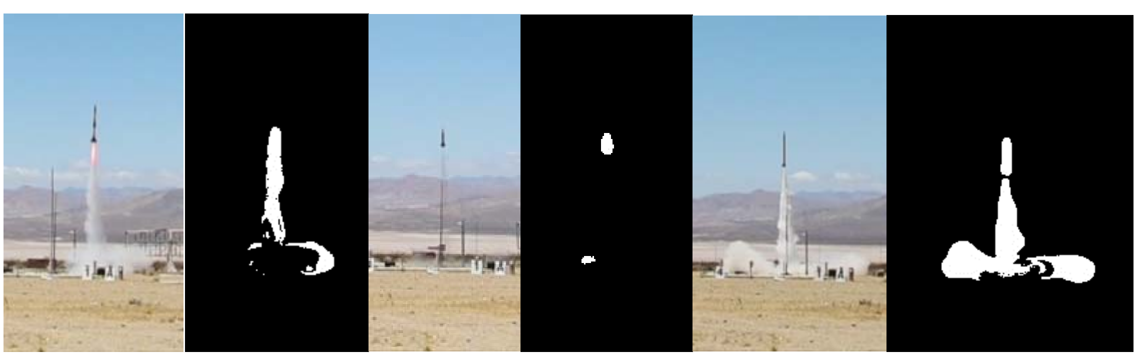

Figure 6. Missile Launches and the Corresponding Background Subtraction Frames

The simulation program determines the size of the smoke plume by calculating the area of the white "blob" in the frame. When the program detects a smoke plume with the required size, a missile launch flag is triggered and the program declares a missile launch.

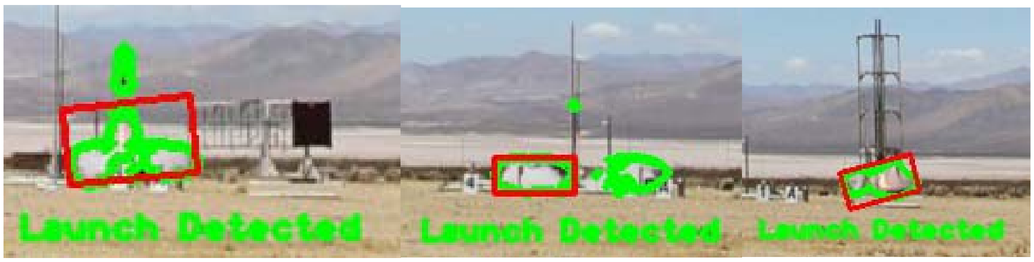

Figure 7. Missile Launch Detected by the Simulation Program

For missiles with minimal smoke plumes, the simulation program can still detect the smoke plumes by setting the threshold to a very low value. However, this may cause the simulation program to falsely declare multiple missile launches for missiles with huge smoke plumes. This is because some missiles eject a lot of smoke before liftoff from the launchpad. This will create a problem for the missile trajectory-tracking phase of the program, as it will not be able to differentiate the missile from the smoke plume or the launch rail. Due to the huge differences in the smoke plume size for different missiles, the threshold level is set at an optimal level where most of the missile launch videos will have a positive missile launch declared. On the other hand, the simulation program will not be able to detect a missile launch for those missiles with minimal smoke plumes.

2. Missile Trajectory-Tracking Phase

The missile trajectory-tracking phase starts only when a missile launch flag is triggered by the missile launch detection phase. To track the missile's trajectory, the simulation program has to detect the position of the missile in each frame. In this case, the background subtraction method is again used to sieve out

the missile moving between each frame. However, the missile location in each frame is not prominent due to the smoke plume at the trailing end of the missile. As the smoke plume is ejecting out of the rocket motor, the background subtraction method detects it as a moving object (Figure 7), causing it to mask the actual missile location. Since the missile location cannot be easily identified from the background subtraction frame, the missile location can still be found by identifying the missile nose tip using the ORB (Oriented FAST and Rotated BRIEF) method.

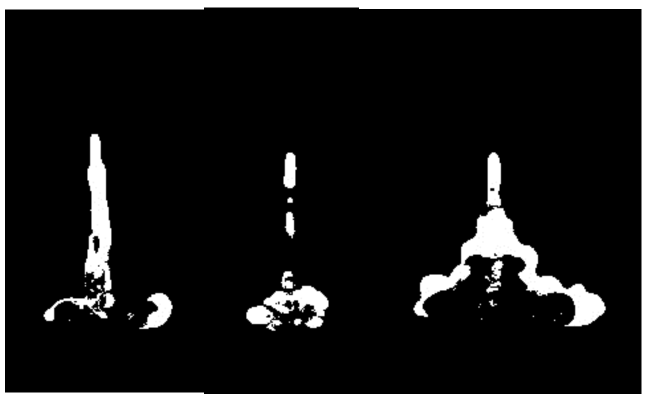

Figure 8. Missile Position Cannot Be Identified after the Background Subtraction Method

a. *Oriented FAST and Rotated BRIEF Method for Determining Missile Location*

As the missile is always located at the tip of the white blob in each frame, the simulation program can determine the missile's location by detecting the leading edge of the white blob using the ORB (Oriented FAST and Rotated BRIEF) method.

The ORB method can detect the leading edge of the white blob, which will in turn determine the missile's nose tip. First, using the method, the algorithm looks for unique features in the "training" and "query" frames[1] and determines the

[1] The "training" frame is usually the immediate frame, or several frames, before the "query" frame. "Training" frames are used as a template for comparison with the "query" frame.

descriptors for each feature in both frames (Figure 9). Thereafter, it compares the feature descriptors of both frames and matches them (Figure 10).

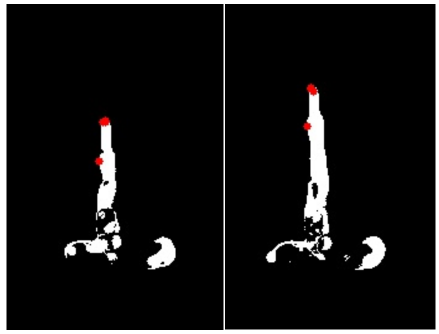

Figure 9. Unique Features detected by the ORB Method on the "Training" Frame (left) and the "Query" Frame (right)

Figure 10. Matching of the Features in the "Training" and "Query" Frames by the ORB Method

Although the ORB method can determine the missile's position, it cannot reliably track the missile's trajectory throughout its flight. This shortcoming arises because the ORB method cannot consistently match the missile nose tip in each frame. As previously described, the ORB method assigns descriptors to determine the missile nose tip, but it may also assign similar descriptors to the smoke plumes that are moving underneath the missile and match them with the missile nose tip. To circumvent this problem, the ORB method must ignore the features detected on the smoke plume and only allow features that are found to be in the same direction of the missile flight. The latter can be used as a match with the missile nose tip in the "training" frame.

The missile flight direction is found by using the coordinates of the first matching missile nose tip in the "training" and "query" frames. With the missile

flight direction found, a search area can be defined in the “query” frame using the position of the missile nose tip, which is found in the “training” frame as the starting point of the search area, as shown in Figure 11.

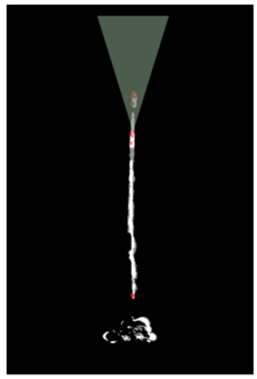

Figure 11. Features Search Area Superimposed on the “Training” and “Query” Frames

If the coordinates of the features found are not in the search area, the simulation program will not include them in the list of features to be used for matching with the “training” frame. In this way, the ORB method would not match the missile nose tip with the smoke plume underneath.

b. Factors Affecting Correct Matching by ORB Method

The method described in the previous section hinges on the first successful match of the missile nose tip in the “training” and “query” frames. Therefore, the first match is very critical in determining the success of the program’s ability to track the missile. To have a good first match, as explained in more detail in the following subsection, the timing to start the missile trajectory-tracking phase is very important. If a missile launch is declared and the missile has not left the launching rail, the missile trajectory-tracking algorithm will not be able to locate the missile, and as result, the program will create an erroneous match. An erroneous match can also occur if the features found on the smoke plume are similar to those found for the missile nose tip of the “training” frame,

even though the ORB method has found the missile nose tip in the "query" frame. This is illustrated in Figure 12 and discussed further at the end of this section.

Figure 12. The Missile Trajectory-Tracking Algorithm Detecting the Smoke Plume instead of the Missile

(1) Delay in Missile Launch from the Launch Rail

From the various missile launch videos, the author observed that different missiles emit different amounts of smoke before they take off and leave the launch rail. By fine tuning the threshold for the size of the smoke plume, we can enable the simulation program to declare a missile launch even when the missile does not leave the launch rail enough for the missile trajectory-tracking algorithm to track it. Furthermore, as different launch rails are used in the videos, the author also noted that the time the missiles took to leave the rail differed. To prevent a false missile tracking, we delayed the missile trajectory-tracking algorithm by a few frames after a missile launch detection flag is triggered. This parameter was then tested on all the missile launch videos to determine the optimal number of frames to be delayed.

(2) Erroneous Matching of Missile Nose Tip

To minimize the erroneous matching of the missile nose tip between the "training" and "query" frames, the missile nose tip features detected on the "query" frame must be prominent for the ORB method to correctly match them

with the corresponding features on the “training” frame. In most of the test cases, when the missile launch detection algorithm declared a missile launch, the ORB method could not correctly match the missile nose tip when the “query” frame was immediately after the “training” frame. This can be due to the insufficient displacement of the missile position between the two frames. To solve this problem, the “query” frame has to be several frames after the “training” frame to allow a good displacement of the missile location between the frames. This parameter was tested on all the missile launch videos to determine the optimal number of frames to be skipped.

Once the simulation program could detect the missile launch and track its trajectory properly, the program was tested with more missile launch videos to determine its performance. These test results are presented in the next chapter.

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III. ALGORITHMS VERIFICATION

Six missile launch videos were available with which to test and verify the performance of the simulation program. As each missile launches differently in the videos, the simulation program was trained with one video and then tested with the other videos to tune the performance of the simulation program. In this way, the performance of the simulation program was optimized for all the missile launch videos.

A. EFFECTS OF CAMERA ORIENTATION

From the logic of the missile trajectory-tracking algorithm, the simulation program is capable of tracking the missile trajectory in all directions. As the missiles were launched from the ground up in all the videos, these videos did not provide any opportunity to test the algorithm against different perspectives on the missile launch direction. Therefore, the missile launch videos are rotated using external video editing software (iMovie) and then tested on the simulation program. A total of three additional missile launch videos were created in this process to simulate the missile launch from top to bottom, left to right, and right to left directions in the video frames.

From Figures 13 and 14, it can be seen that the simulation program is capable of tracking the missile trajectory in the horizontal and vertical directions. Therefore, this test proved that the simulation program is able to track a missile launch in any direction.

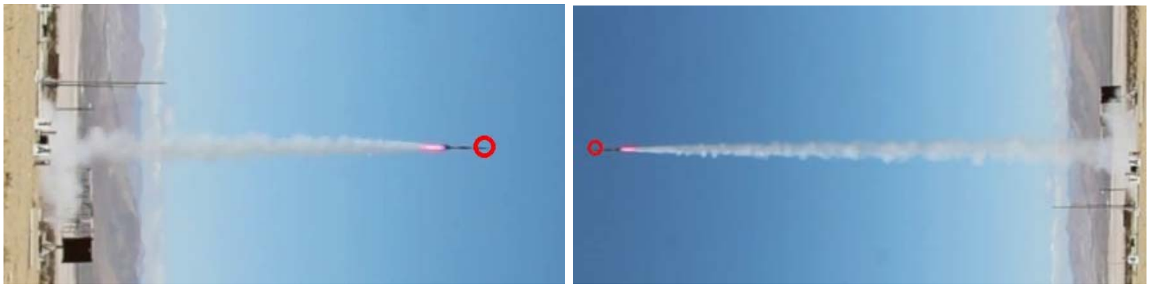


Figure 13. Testing of the Simulation Program for Missile Launch in Horizontal Direction

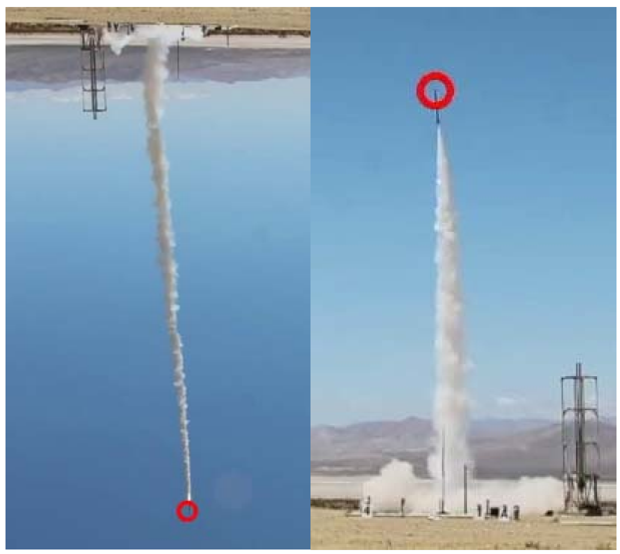


Figure 14. Testing of the Simulation Program for Missile Launch in Vertical Direction

B. ALGORITHM INITIATION TUNING

As mentioned in the previous chapter, there are two parameters in the simulation program that need to be optimized to improve the performance of the simulation program. They are: 1) the number of frames to be delayed after the missile launch flag has been triggered, and 2) the number of frames to skip after the “training” frame to select the “query” frame. In this optimization test, a total of nine missile launch videos are used, including the additional three missile launch videos mentioned in the previous section.

The optimization tests are done by running all the videos through the simulation program and evaluating the performance of the program on individual frames in each video. The evaluation criteria for each frame are categorized as follows:

- True Positive: The simulation program correctly identifies the missile location in the frame.
- False Alarm (False Positive): The simulation program wrongly identifies another object as the missile location in the frame.
- False Negative: The simulation program does not identify the missile location in the frame when the missile is in flight.

Once all the frames have been evaluated, the total number of frames for each category is tallied to calculate the average percentage of true positives and false alarms. This determines the performance of the simulation program for each combination of the two parameters.

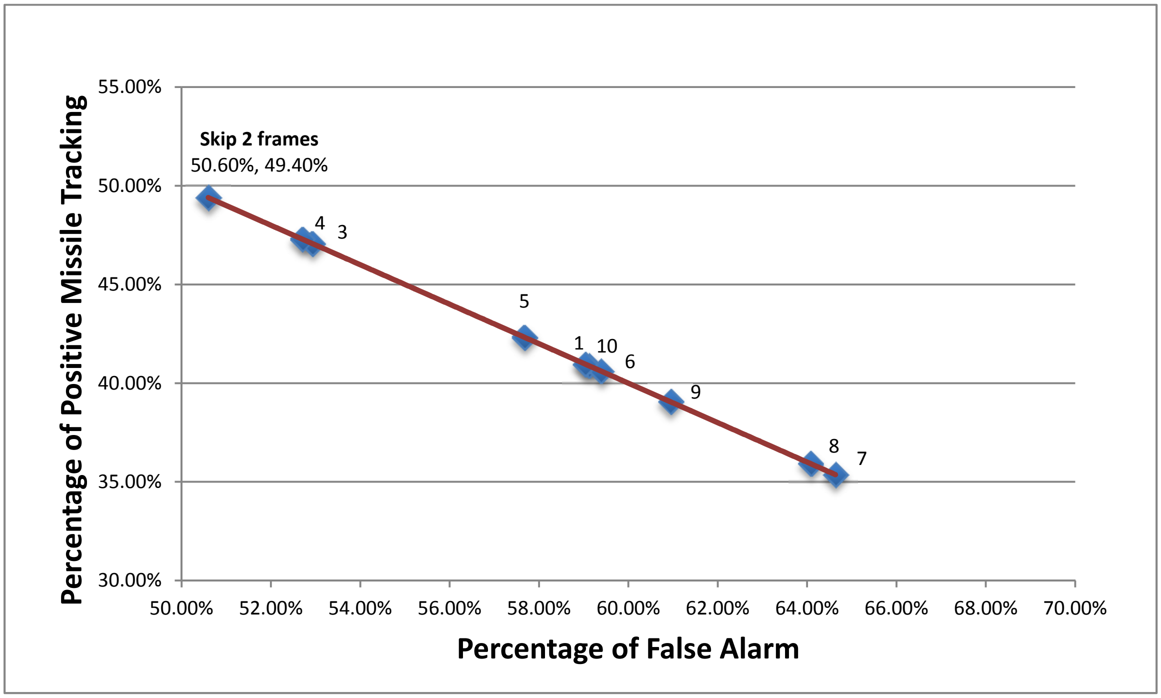
As there are two parameters to be optimized, one of the parameters has to be fixed while the other parameter is iterated. For the first optimization test, the number of frames to be delayed after missile launch flag triggered is set at four frames, while the number of frames to be skipped after the "Training" frame is iterated from one to ten frames. In the second optimization test, the optimal number of frames to be skipped, found in the first test, is used while iterating the number of frames to be delayed after missile launch, from one to ten frames.

1. Number of Frames to Skip after the "Training" Frame

The results of the first optimization test are summarized in Table 1 and plotted in Figure 14.

Table 1. Test Results of the First Optimization Test

Number of frames to skip (9 videos)	1	2	3	4	5	6	7	8	9	10
Average % of true positive tracking	40.96	49.40	47.30	47.08	42.33	40.61	35.36	35.92	39.06	40.88
Average % of false alarm	59.04	50.60	52.70	52.92	57.67	59.39	64.64	64.08	60.94	59.12



The number by each marker represents the number of frames that were skipped.

Figure 15. Outcome of the First Optimization Test

From Figure 15, it follows that the optimal number of frames to be skipped is two frames. This is because the simulation program has the highest true positive percentage[2] at 49.4 percent and the lowest false alarm rate at 50.6 percent.

2 The percentage calculation is based on the total number of frames for all nine tested videos.

If the two missile launch videos with minimal smoke plume are excluded from the test, the test results have an improvement of about 10 percent in both the true positive percentage and false alarm rate. The newly tabulated test results are summarized in Table 2.

Table 2. Test Results of the First Optimization Test without Minimal Smoke Plume Missile Launch Videos

Number of frames to skip (7 videos)	1	2	3	4	5	6	7	8	9	10
Average % of true positive tracking	50.10	59.14	58.36	58.09	49.90	49.87	40.69	46.18	48.63	52.56
Average % of false alarm	49.90	40.86	41.64	41.91	50.10	50.13	59.31	53.82	51.37	47.44

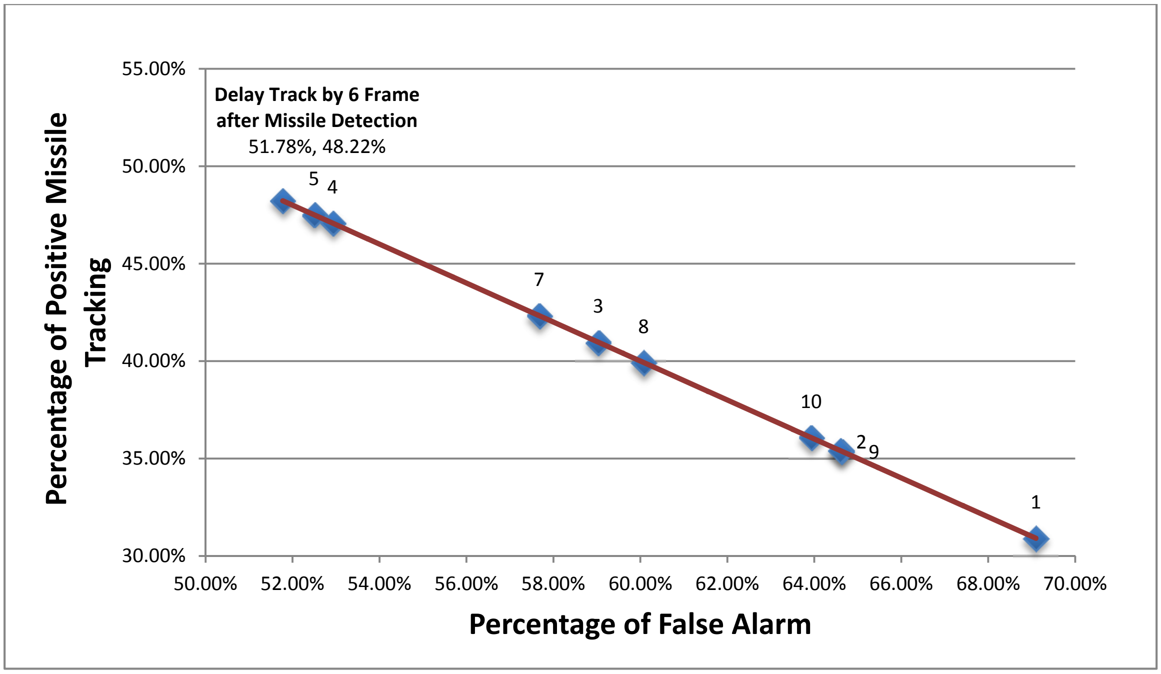
From Table 2, the optimal number of frames to skip is still at two frames even though the missile launch videos with minimal smoke plume are excluded from the tabulation. This shows that the missile launch videos with minimal smoke plumes do not have any impact on the optimization test.

2. Number of Frames Delayed after Missile Launch Flag Triggered

From the first test results, the optimized number of frames to be skipped from the 'training' frame is used in the second test to find the optimized number of frames to be delayed. The results are summarized in Table 3 and plotted in Figure 15.

Table 3. Test Results of the Second Optimization Test

Number of frames delayed (9 videos)	1	2	3	4	5	6	7	8	9	10
Average % of true positive tracking	30.90	35.41	40.96	47.08	47.51	48.22	42.33	39.93	35.36	36.07
Average % of false alarm	69.10	64.59	59.04	52.92	52.49	51.78	57.67	60.07	64.64	63.93



The number by each marker represents the number of frames that were delayed.

Figure 16. Visualization of the Second Optimization Test Results

The second optimization test results show that when the simulation program starts tracking the missile at the sixth frame after the missile launch flag was triggered, it would have the highest true positive percentage at 48.22 percent and the lowest false alarm rate at 51.78 percent.

When the two missile launch videos with minimal smoke plume are excluded from the test, the test results have an improvement of about 3 percent to 10 percent in both the true positive percentage and false alarm rate. The newly tabulated test results are shown in Table 4.

Table 4. Test Results of the Second Optimization Test without Minimal Smoke Plume Missile Launch Videos.

Number of frames delayed (7 Videos)	1	2	3	4	5	6	7	8	9	10
Average % of true positive tracking	33.87	40.44	50.10	58.09	58.64	57.52	49.90	48.98	40.69	46.18
Average % of false alarm	66.13	59.56	49.90	41.91	41.36	42.48	50.10	51.02	59.31	53.82

The optimal number of frames to be delayed after the missile launch flag triggered has changed from the sixth frame to the fifth frame. By comparing the results of the optimal frames in Tables 3 and 4, we find a 10 percent improvement in both the true positive tracking percentage and false alarm rate. This shows that the missile videos with minimal smoke plumes have a significant influence on this parameter. The performance of the simulation program is improved from a percentage of 48.22 percent in true positive tracking and 51.78 percent false alarm rate, to 58.64 percent in true positive tracking and 41.36 percent in false alarm rate.

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IV. POSITION ESTIMATION

A. MISSILE SPEED

As mentioned previously, military operations are making increased use of UAVs, and the purpose of this thesis is to explore the feasibility of their use to patrol a potential ICBM launch area. The onboard camera of such a UAV has the capability to record the tilt and pan of a gimbal, which, along with inertial measurement unit/ global positioning system (IMU/GPS) information, allows calculating a missile's geographic position. Figure 17 represents a general geometry of tracking a missile launch by a UAV.

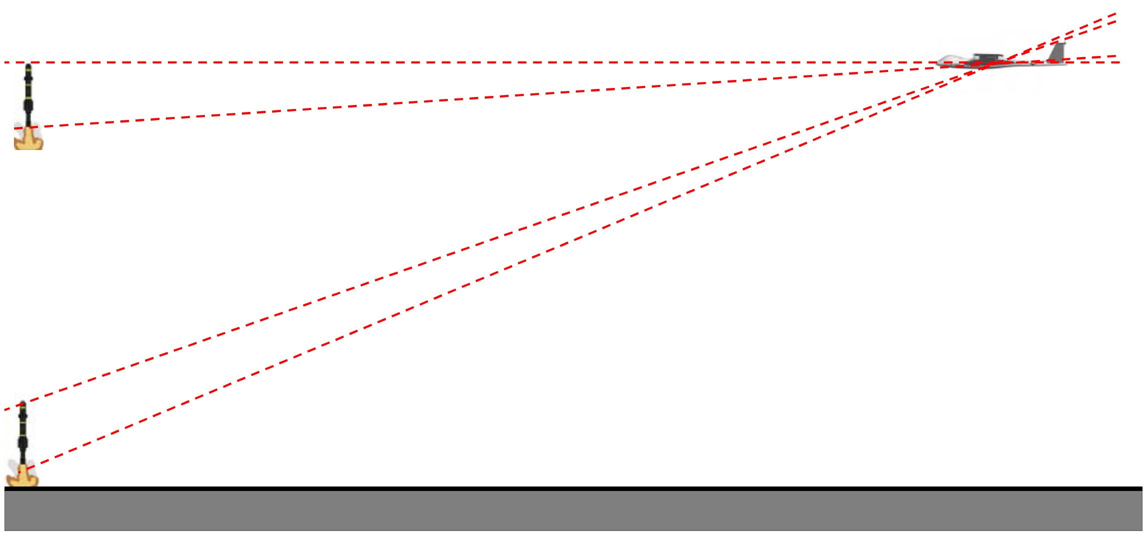

Figure 17. Observing a Missile Launch from UAV

In the experiment that took place in Mojave Desert, there was no tilt/pan information (the stationary ground camera was used) and the camera's position data was not available either. Hence, only a limited analysis on the missile could be carried over. Figure 18 represents the geometry in this case.

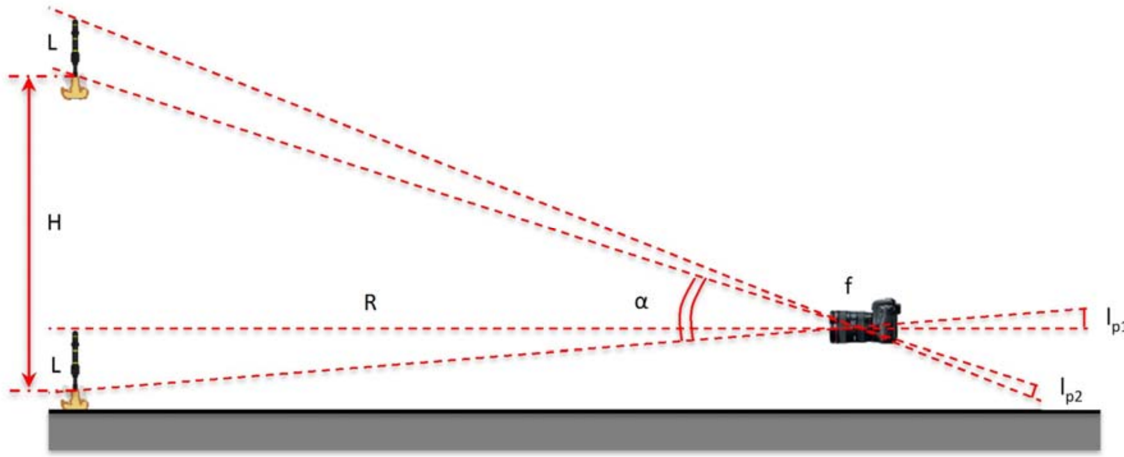

Figure 18. Detecting a Missile Launch using the Pinhole Camera Model

Using the pinhole camera model, the size of a rocket in the image frame, l_p via its physical size, L, can be expressed as

$$l_p = \frac{f}{R}L = \frac{1}{k}L \tag{1}$$

In Equation (1), R is the (unknown) range to the missile launch site and f is the (unknown) focal point of the camera. With the values L and l_p known before the launch, the coefficient k can be estimated as

$$k = \frac{L}{l_p} \text{ [m/pixel]} \tag{2}$$

Therefore, the rocket speed, v, can be estimated using the camera frame rate, η, and the number of pixels the rocket has moved between the two consecutive frames, d_p , as

$$v = kd_p\eta \text{ [m/s]} \tag{3}$$

Figure 18 shows how the estimated rocket speed calculated from the simulation program is displayed in the video.

Figure 19. Rocket Speed Calculation

Figure 19 provides a comparison of the rocket speed calculated by the simulation program and the actual speed recorded using the accelerometer and barometric altimeter carried onboard the rocket. The errors in the speed values are due to the simulation program limited accuracy in detecting the rocket position. The simulation program cannot accurately detect the same spot on the moving rocket in each frame.

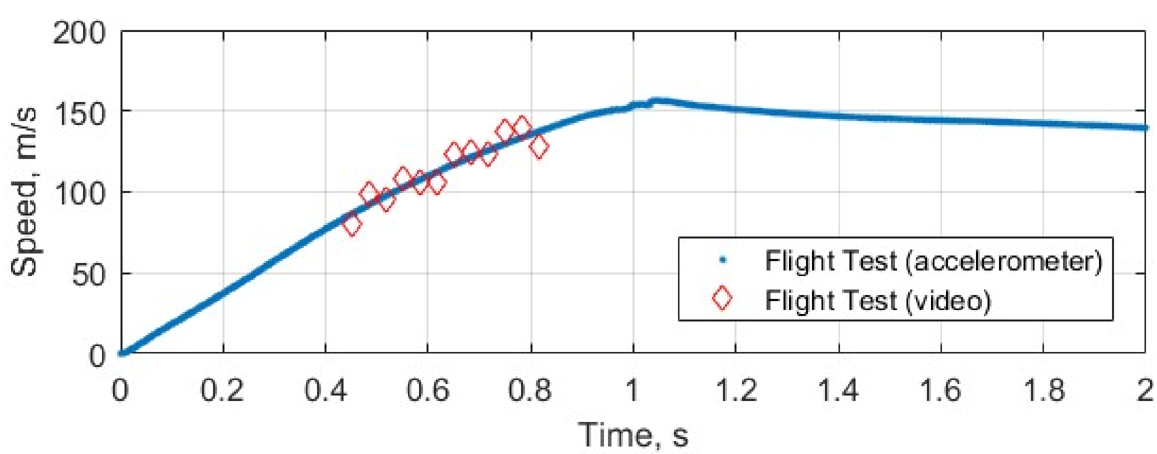

Figure 20. Rocket Speed Time History

The aforementioned method of calculating the rocket speed could be enhanced by accounting for the elevation angle α (see Figure 17). In this case, Equation (3) becomes

$$v = \frac{k d_p \eta}{\cos\alpha} \text{ [m/s]} \tag{4}$$

where $\alpha = \tan^{-1}\left(\frac{H}{R}\right)$. While the rocket altitude, H, could be estimated as

$$H = \int_0^t v\,dt \tag{5}$$

(and verified using onboard sensor data), the range from the camera focal point to the missile launch site was not measured. In addition, Equation (4) does not account for the rocket orientation, which might be slightly different from a straight vertical.

Another set of field experiments to gather more data and conduct an advanced analysis was scheduled in mid-August, but it was postponed. Hence, the results based on those additional trials did not make it into this thesis.

B. MISSILE ORIENTATION

Using two consecutive frames of a stationary camera, a launch plane orientation (with respect to the camera coordinate frame) can also be determined as

$$\theta = \tan^{-1}\left(\frac{y_2 - y_1}{x_2 - x_1}\right) \tag{6}$$

where (x_1, y_1) refers to the coordinates of the missile nose tip in the first frame and (x_2, y_2) refers to the coordinates of the missile nose tip in the second frame.

The deflection angle (from the vertical), computed using Equation (6), by the simulation program is shown in Figure 20.

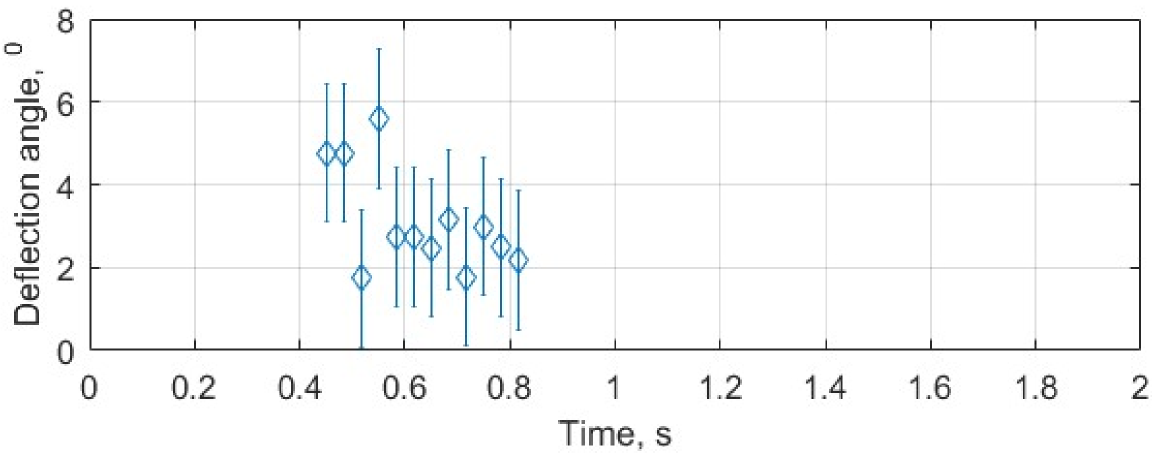

Figure 21. Time History of Missile Deflection Angle

The error bars shown in Figure 20 are estimated based on the rocket speed and the fact that x- and y-coordinates of the rocket nose tip are only accurate to about 1 pixel. These outliers are probably related to the fact that the camera was mounted on a tripod relatively close to the launch position, causing it to vibrate during the launch.

If a missile launch is observed by two UAVs, using the plane angle θ would result in an estimate of the missile altitude, which would in turn result in a further correction of Equation (4).

C. SUMMARY

Based on the results of the calculated missile speed by the simulation program, the program showed that it is capable of estimating the missile speed using the missile position captured in each frame. The simulation program is also able to calculate the missile deflection angle from the vertical. However, it will only be useful when the missile deflection angle is calculated by two UAVs separated by at least 90 degrees in azimuth from the missile launch site. In this way, the missile altitude can be triangulated using the plane angles from each UAV.

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V. CONCLUSION AND FUTURE RESEARCH

A. CONCLUSION

The simulation program was able to detect a missile launch and track its trajectory in the video's field of view. Although it could only achieve a true positive tracking percentage of 58.46 percent, the update rate of the missile position by the simulation program was sufficient for a weapon system to intercept it reliably. This means that for a missile launch video with 25 frames per second, the simulation program was able to provide about 14 updates on the missile position per second.

On the other hand, the simulation program is unable to process any missile launch videos that are not stabilized and is also unable to detect a missile launch of a missile with a minimal smoke plume. In its current state, the simulation program cannot process the surveillance videos from the CUAV to achieve its objective of providing targeting information for an attacking UAV to reliably intercept it. However, if the surveillance video can be stabilized before processed by the simulation program, it can be installed on a surveillance UAV to detect a missile launch threat and provide targeting information to an attacking UAV to reliably intercept the missile.

B. FURTHER STUDIES

The simulation program can be enhanced to improve the missile trajectory tracking methods and determine the 3D location of the missile. This information will enhance the accuracy of the targeting information that will be fed to the attacking UAV for intercepting the missile. Further work can also be done to stabilize the video in real time before processing the missile launch video using the simulation program. This will allow real-time surveillance by the UAV at the missile launch site.

1. Determine Missile Location in 3D Space

The calculation of the missile speed and orientation by the current simulation program is only accurate if the missile down range of the video camera remains constant, which will not be possible in real life. To determine the actual 3D location of the missile, future research should use multiple cameras with known distances and bearing from each other, to enable the calculation of the missile location using the triangulation method [11]. By using the missile trajectory-tracking algorithm in this simulation program, the researcher can write a program to process the missile launch videos from multiple cameras and determine a missile's location in 3D space.

2. Object Motion Tracking

The background subtraction method used in this simulation program is unable to identify the missile position in each frame due to the smoke plume created by the missile rocket motor. In addition, the missile launch algorithm of the simulation program is also unable to detect the launch of a missile with a minimal smoke plume.

Instead of using the background subtraction method to track object motion, researchers can consider the optical flow technique. Optical flow [12] performs estimation of the motion in each individual pixel of the frame by calculating the motion of each pixel. Once the motion of each pixel has been calculated, the mean shift method can be used to cluster the pixels with similar motion descriptions together to identify the missile's location in each frame.

3. Video Stabilization

To perform detection of a missile launch and trajectory tracking from a single or multiple CUAVs, the researcher must ensure the video footage from the CUAV is stabilized before any video processing can be done. Matsushita et al. recommended a methodology to stabilize a video with minimal impact to the size of the video frames [13]. This methodology can be applied to stabilize the missile

launch video and subsequently feed it into the simulation program for missile launch detection and trajectory tracking. In this way, the videos taken from the CUAV(s) can be used to perform a surveillance mission and track a launched missile so that an attacking UAV can intercept it.

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APPENDIX A. SIMULATION PROGRAM PYTHON CODE

The Python code provided here detects the missile launch from a video and tracks the missile's trajectory within the video's field of view. When a missile launch is detected by the simulation program, a "Missile Launch Detected" indicator appears in the top left corner of the video. The speed and orientation of the missile launch are indicated in the same corner. A green indicator flashes on the video to indicate a missile launch detected from the missile smoke plume. If the simulation program also detects the missile's location, a red circle shows up in the video to indicate that location. Finally, the simulation program overlays all indicators mentioned previously on the existing missile launch video and outputs it as a video file saved on the hard disk.

```
import sys
sys.path.append('/Applications/OpenCV3/build/lib/python3')

import cv2
import numpy as np
import matplotlib.pyplot as plt
import math
cv2.ocl.setUseOpenCL(False)

# Loading a missile launch video
cap = cv2.VideoCapture('ML-Side1.mov')
framecount = 1

#Initialise the background subtraction technique
bgs = cv2.createBackgroundSubtractorMOG2()

#Retrieve the video properties
fps = cap.get(cv2.CAP_PROP_FPS)
width = cap.get(cv2.CAP_PROP_FRAME_WIDTH)
height = cap.get(cv2.CAP_PROP_FRAME_HEIGHT)
count = cap.get(cv2.CAP_PROP_FRAME_COUNT)
outsize = (int(width), int(height))
TFrame = cap.get(7)

#Initialise the parameters to output a video
```

```
fourcc = cv2.VideoWriter_fourcc(*'XVID')
out = cv2.VideoWriter('ML1 Final.avi', fourcc, fps, outsize, isColor=True)

a = 0
b = 0
Ang1 = 0
flag=1
delay=0
detection =0
firstorb = 0
FSkip = 1000000
MLcount = 100000
deFlag = 0
LText = 1
Repeat = 0
MxVel = []

#Initialise count for performance verification
Positive = 0
FPositive = 0
DCount = 0

while True and framecount <= TFrame:
   ret, frame = cap.read()
   Detect = frame.copy()
   print('\n\nFrame count = ',framecount)

   if frame is None :
      print('\nFrame is empty!!!!')
      break

   #Perform Gaussian Blur to the frame before background subtraction
   gframe = cv2.GaussianBlur(frame, (15, 15), 0)
   fgmask = bgs.apply(gframe)
   ret,thresh = cv2.threshold(fgmask,125,255,cv2.THRESH_BINARY)

   if framecount ==2:
      M1 = thresh.copy()

   M2 = thresh.copy()

   if framecount >2:
      im2, contours, hierarchy =
cv2.findContours(thresh,cv2.RETR_LIST,cv2.CHAIN_APPROX_SIMPLE)
      moments = [cv2.moments(x) for x in contours]
```

```
        #Calculation of the missile smoke plume size for missile launch detection
        for c in contours:
            area = cv2.contourArea(c)
            #print('Area is ', area, '\n')
            if(area>500) and deFlag == 0:
                detection=1
                print('Launch Detected!!!!')
                MLcount = framecount + 6
                cv2.drawContours(Detect, c, -1, (0,255,0), 3)
                deFlag = 1
        #Creation of the Missile Launch indicator text
        if detection == 1:
            font = cv2.FONT_HERSHEY_SIMPLEX
            LText +=1
            if LText %2 == 0:
                cv2.putText(Detect,'Missile Launch Detected',(100,100), font,
1,(50,50,255),3,cv2.LINE_AA)

    print('Checking flag = ', framecount >= MLcount)

    #Feature Matching Algorithm
    if framecount >= MLcount:

        orb = cv2.ORB_create(nfeatures=200)
        kps1 = orb.detect(M1, None)
        kps1, descriptors1 = orb.compute(M1, kps1)

        if len(kps1)!= 0 and framecount <= FSkip:
            if firstorb == 0:
                #Number of frame to skip after the "Training" frame
                FSkip = framecount + 2
                firstorb = 1
        else:
            DCount += 1
            kps2 = orb.detect(M2, None)
            kps2, descriptors2 = orb.compute(M2, kps2)
            bf = cv2.BFMatcher(cv2.NORM_HAMMING, crossCheck=True)

            if len(kps2) != 0:
                matches = bf.match(descriptors1, descriptors2)
                matchesy = sorted(matches, key = lambda y:y.distance)
                print("Found {} matching feature pairs.".format(len(matches)))
                Best = matchesy[:5]
```

```
if len(matches) != 0:

    # Initialize lists of matching fetures
    SelectTrans = 0
    EuclideanDist = []
    Px1 = []
    Py1 = []
    Px2 = []
    Py2 = []
    AngList = []
    i = 0

    # For each match...
    for mat in Best:

        # Get the matching keypoints for each frame
        img1_idx = mat.queryIdx
        img2_idx = mat.trainIdx

        # x - columns
        # y - rows
        # Get the coordinates
        (x1,y1) = kps1[img1_idx].pt
        (x2,y2) = kps2[img2_idx].pt
        Px1.append(x1)
        Py1.append(y1)
        Px2.append(x2)
        Py2.append(y2)

        DistTrans = math.sqrt(math.pow((x1-x2),2)+math.pow((y1-y2),2))
        EuclideanDist.append(DistTrans)

    #Initial Parameters for the first iteration
    SEu = np.argmax(EuclideanDist, axis=0)

    if a == 0 and b == 0:
        a = Px1[SEu]
        b = Py1[SEu]

    #Calculation of the missile orientation
    Ang = -math.atan2((Py2[SEu]-b),(Px2[SEu]-a))

    if Ang1 == 0:
        Ang1 = Ang
```

```
            #Checking whether the features found are within the search area.
            if Ang <= (Ang1 + math.radians(35)) and Ang >= (Ang1 -
math.radians(35)): #y2 - b < 0:
               Olda = a
               Oldb = b
               a = Px2[SEu]
               b = Py2[SEu]
               Ang1 = Ang

            DistTrans = 0

            if b != b1:
               a = np.int0(a)
               b = np.int0(b)

               #Draw a circle on the detected missile location
               cv2.circle(Detect,(a,b), 10,(0,0,255),3)
               print('Circle Drawn\n')

               #Calculate the distance that the missile has travelled between 2
frames
               if Repeat == 0:
                  MxDist = math.sqrt(math.pow((a-Olda),2)+math.pow((b-
Oldb),2))
               else:
                  MxDist = math.sqrt(math.pow((Py2[SEu]-
Py1[SEu]),2)+math.pow((Px2[SEu]-Px1[SEu]),2))
                  Repeat = 0

               #Calculate the speed of the missile
               MxSpeed = np.around(((MxDist/22.601)*1.52)*fps, decimals=2)
               print('\nMissile Speed = ', MxSpeed)

               #Display of the missile speed and orientation in the output video
               cv2.putText(Detect,'Missile Speed = %.2f m/s' %
MxSpeed,(100,150), font, 1,(50,255,50),2,cv2.LINE_AA)
               cv2.putText(Detect,'Deflection Angle = %.2f degrees' %
np.around(90 - math.degrees(Ang), decimals=2),(100,200), font,
1,(50,255,50),2,cv2.LINE_AA)
               cv2.putText(Detect,'from the Vertical',(100,250), font,
1,(50,255,50),2,cv2.LINE_AA)
               MxVel.append(format(MxSpeed, '.2f'))
               Positive += 1
               print('Deflection Angle = ', 90 - math.degrees(Ang))
            else:
```

```
                    cv2.putText(Detect,'Missile Speed = %.2f m/s' %
MxSpeed,(100,150), font, 1,(50,255,50),2,cv2.LINE_AA)
                    cv2.putText(Detect,'Deflection Angle = %.2f degrees' %
np.around(90 - math.degrees(Ang1), decimals=2),(100,200), font,
1,(50,255,50),2,cv2.LINE_AA)
                    cv2.putText(Detect,'from the Vertical',(100,250), font,
1,(50,255,50),2,cv2.LINE_AA)
                    Repeat = 1
                    FPositive += 1
                    print('\nMissile Speed = ', MxSpeed)
                    print('Deflection Angle = ', 90 - math.degrees(Ang1))

          else:
             print('No matches in kps2')

   M1 = M2.copy()
   b1 = b

   #Output the processed video
   out.write(Detect)

   #Display the processed video
   cv2.imshow('Missile Launch Post processing', Detect)
   cv2.waitKey(100)

   if cv2.waitKey(1) & 0xFF == ord ('q'):
      break
   framecount += 1

print('\nThe number of postive detection is ', Positive)
print('The number of false postive detection is ', FPositive)
print('The expected total number of detection is ', DCount)
print('The number of frames true negative in the video is ', (framecount -
DCount))
print('\nThe camera fps is ', fps)
cap.release()
cv2.destroyAllWindows()
```

APPENDIX B. TEST RESULTS OF MISSILE LAUNCH VIDEOS

A. OPTIMIZATION TEST RESULT FOR THE NUMBER OF FRAMES TO SKIP AFTER THE "TRAINING" FRAME

Skip Frame = 10	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	7	17	21	9	0	0	8	7	8
False Positive	7	11	10	9	7	66	7	8	9
False Negative	0	0	0	1	1	3	0	0	2
False Alarm	7	11	10	10	8	69	7	8	11
Total	14	28	31	19	8	69	15	15	19
% Positive	50.00%	60.71%	67.74%	47.37%	0.00%	0.00%	53.33%	46.67%	42.11%
% False Alarm	50.00%	39.29%	32.26%	52.63%	100.00%	100.00%	46.67%	53.33%	57.89%
Average % Positive	40.88%	Average % Positive w/o ML5 & ML2-Slow				52.56%			
Average % False Alarm	59.12%	Average % False Alarm w/o ML5 & ML2-Slow				47.44%			

Skip Frame = 9	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	8	18	22	10	1	0	9	8	0
False Positive	7	11	10	9	8	68	7	8	19
False Negative	0	0	0	1	0	2	0	0	1
False Alarm	7	11	10	10	8	70	7	8	20
Total	15	29	32	20	9	70	16	16	20
% Positive	53.33%	62.07%	68.75%	50.00%	11.11%	0.00%	56.25%	50.00%	0.00%
% False Alarm	46.67%	37.93%	31.25%	50.00%	88.89%	100.00%	43.75%	50.00%	100.00%
Average % Positive	39.06%	Average % Positive w/o ML5 & ML2-Slow				48.63%			
Average % False Alarm	60.94%	Average % False Alarm w/o ML5 & ML2-Slow				51.37%			

Skip Frame = 8	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D

Positive	9	18	0	11	0	0	10	9	9
False Positive	7	11	30	9	9	68	7	8	10
False Negative	0	1	3	1	1	3	0	0	2
False Alarm	7	12	33	10	10	71	7	8	12
Total	16	30	33	21	10	71	17	17	21
% Positive	56.25%	60.00%	0.00%	52.38%	0.00%	0.00%	58.82%	52.94%	42.86%
% False Alarm	43.75%	40.00%	100.00%	47.62%	100.00%	100.00%	41.18%	47.06%	57.14%
Average % Positive	35.92%				Average % Positive w/o ML5 & ML2-Slow	46.18%			
Average % False Alarm	64.08%				Average % False Alarm w/o ML5 & ML2-Slow	53.82%			

Skip Frame = 7	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	10	19	22	12	2	11	0	0	10
False Positive	7	12	11	9	9	59	17	17	10
False Negative	0	0	1	1	0	2	1	1	2
False Alarm	7	12	12	10	9	61	18	18	12
Total	17	31	34	22	11	72	18	18	22
% Positive	58.82%	61.29%	64.71%	54.55%	18.18%	15.28%	0.00%	0.00%	45.45%
% False Alarm	41.18%	38.71%	35.29%	45.45%	81.82%	84.72%	100.00%	100.00%	54.55%
Average % Positive	35.36%				Average % Positive w/o ML5 & ML2-Slow	40.69%			
Average % False Alarm	64.64%				Average % False Alarm w/o ML5 & ML2-Slow	59.31%			

Skip Frame = 6	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	10	19	23	12	0	12	11	2	11
False Positive	7	12	12	9	10	59	8	17	10
False Negative	1	1	0	2	2	2	0	0	2
False Alarm	8	13	12	11	12	61	8	17	12
Total	18	32	35	23	12	73	19	19	23
% Positive	55.56%	59.38%	65.71%	52.17%	0.00%	16.44%	57.89%	10.53%	47.83%
% False Alarm	44.44%	40.63%	34.29%	47.83%	100.00%	83.56%	42.11%	89.47%	52.17%
Average % Positive	40.61%	Average % Positive w/o ML5 & ML2-Slow				49.87%			
Average % False Alarm	59.39%	Average % False Alarm w/o ML5 & ML2-Slow				50.13%			

Skip Frame = 5	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	11	20	24	13	2	12	12	10	0
False Positive	8	13	12	9	10	60	8	10	23
False Negative	0	0	0	2	1	2	0	0	1
False Alarm	8	13	12	11	11	62	8	10	24
Total	19	33	36	24	13	74	20	20	24
% Positive	57.89%	60.61%	66.67%	54.17%	15.38%	16.22%	60.00%	50.00%	0.00%
% False Alarm	42.11%	39.39%	33.33%	45.83%	84.62%	83.78%	40.00%	50.00%	100.00%
Average % Positive	42.33%	Average % Positive w/o ML5 & ML2-Slow				49.90%			
Average % False Alarm	57.67%	Average % False Alarm w/o ML5 & ML2-Slow				50.10%			

Skip Frame = 4	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	12	22	26	14	0	13	14	12	12
False Positive	9	13	12	9	13	61	8	10	11
False Negative	0	0	0	3	2	2	0	0	3
False Alarm	9	13	12	12	15	63	8	10	14
Total	21	35	38	26	15	76	22	22	26
% Positive	57.14%	62.86%	68.42%	53.85%	0.00%	17.11%	63.64%	54.55%	46.15%
% False Alarm	42.86%	37.14%	31.58%	46.15%	100.00%	82.89%	36.36%	45.45%	53.85%
Average % Positive	47.08%	Average % Positive w/o ML5 & ML2-Slow				58.09%			
Average % False Alarm	52.92%	Average % False Alarm w/o ML5 & ML2-Slow				41.91%			

Skip Frame = 3	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	13	22	26	15	0	13	14	12	11
False Positive	9	13	12	9	13	61	8	10	11
False Negative	0	0	0	2	2	2	0	0	4
False Alarm	9	13	12	11	15	63	8	10	15
Total	22	35	38	26	15	76	22	22	26
% Positive	59.09%	62.86%	68.42%	57.69%	0.00%	17.11%	63.64%	54.55%	42.31%
% False Alarm	40.91%	37.14%	31.58%	42.31%	100.00%	82.89%	36.36%	45.45%	57.69%
Average % Positive	47.30%	Average % Positive w/o ML5 & ML2-Slow				58.36%			
Average % False Alarm	52.70%	Average % False Alarm w/o ML5 & ML2-Slow				41.64%			

Skip Frame = 2	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	13	23	27	15	2	14	15	13	12
False Positive	9	13	12	9	11	61	8	10	11
False Negative	0	0	0	3	3	2	0	0	4
False Alarm	9	13	12	12	14	63	8	10	15
Total	22	36	39	27	16	77	23	23	27
% Positive	59.09%	63.89%	69.23%	55.56%	12.50%	18.18%	65.22%	56.52%	44.44%
% False Alarm	40.91%	36.11%	30.77%	44.44%	87.50%	81.82%	34.78%	43.48%	55.56%
Average % Positive	49.40%	Average % Positive w/o ML5 & ML2-Slow				59.14%			
Average % False Alarm	50.60%	Average % False Alarm w/o ML5 & ML2-Slow				40.86%			

Skip Frame = 1	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	14	24	0	15	0	14	16	14	13
False Positive	9	13	37	9	16	62	8	10	11
False Negative	0	0	3	4	1	2	0	0	4
False Alarm	9	13	40	13	17	64	8	10	15
Total	23	37	40	28	17	78	24	24	28
% Positive	60.87%	64.86%	0.00%	53.57%	0.00%	17.95%	66.67%	58.33%	46.43%
% False Alarm	39.13%	35.14%	100.00%	46.43%	100.00%	82.05%	33.33%	41.67%	53.57%
Average % Positive	40.96%	Average % Positive w/o ML5 & ML2-Slow				50.10%			
Average % False Alarm	59.04%	Average % False Alarm w/o ML5 & ML2-Slow				49.90%			

B. OPTIMIZATION TEST RESULT FOR THE NUMBER OF FRAMES TO DELAY AFTER MISSILE LAUNCH

Start Track = 10	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	9	18	0	11	0	1	10	9	9
False Positive	7	11	30	9	9	68	7	8	10
False Negative	0	1	3	1	1	2	0	0	2
False Alarm	7	12	33	10	10	70	7	8	12
Total	16	30	33	21	10	71	17	17	21
% Positive	56.25%	60.00%	0.00%	52.38%	0.00%	1.41%	58.82%	52.94%	42.86%
% False Alarm	43.75%	40.00%	100.00%	47.62%	100.00%	98.59%	41.18%	47.06%	57.14%
% False Positive	43.75%	36.67%	90.91%	42.86%	90.00%	95.77%	41.18%	47.06%	47.62%
Average % Positive	36.07%	Average % Positive w/o ML5 & ML2-Slow				46.18%			
Average % False Alarm	63.93%	Average % False Alarm w/o ML5 & ML2-Slow				53.82%			

Start Track = 9	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	10	19	22	12	2	11	0	0	10
False Positive	7	12	11	9	9	59	17	17	10
False Negative	0	0	1	1	0	2	1	1	2
False Alarm	7	12	12	10	9	61	18	18	12
Total	17	31	34	22	11	72	18	18	22
% Positive	58.82%	61.29%	64.71%	54.55%	18.18%	15.28%	0.00%	0.00%	45.45%
% False Alarm	41.18%	38.71%	35.29%	45.45%	81.82%	84.72%	100.00%	100.00%	54.55%
% False Positive	41.18%	38.71%	32.35%	40.91%	81.82%	81.94%	94.44%	94.44%	45.45%
Average % Positive	35.36%	Average % Positive w/o ML5 & ML2-Slow				40.69%			
Average % False Alarm	64.64%	Average % False Alarm w/o ML5 & ML2-Slow				59.31%			

Start Track = 8	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	10	19	23	13	0	12	11	0	11
False Positive	7	12	12	9	10	59	8	17	10
False Negative	1	1	0	1	2	2	0	2	2
False Alarm	8	13	12	10	12	61	8	19	12
Total	18	32	35	23	12	73	19	19	23
% Positive	55.56%	59.38%	65.71%	56.52%	0.00%	16.44%	57.89%	0.00%	47.83%
% False Alarm	44.44%	40.63%	34.29%	43.48%	100.00%	83.56%	42.11%	100.00%	52.17%
% False Positive	38.89%	37.50%	34.29%	39.13%	83.33%	80.82%	42.11%	89.47%	43.48%
Average % Positive	39.93%	Average % Positive w/o ML5 & ML2-Slow				48.98%			
Average % False Alarm	60.07%	Average % False Alarm w/o ML5 & ML2-Slow				51.02%			

Start Track = 7	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	11	20	24	13	2	12	12	10	0
False Positive	8	13	12	9	10	60	8	10	23
False Negative	0	0	0	2	1	2	0	0	1
False Alarm	8	13	12	11	11	62	8	10	24
Total	19	33	36	24	13	74	20	20	24
% Positive	57.89%	60.61%	66.67%	54.17%	15.38%	16.22%	60.00%	50.00%	0.00%
% False Alarm	42.11%	39.39%	33.33%	45.83%	84.62%	83.78%	40.00%	50.00%	100.00%
% False Positive	42.11%	39.39%	33.33%	37.50%	76.92%	81.08%	40.00%	50.00%	95.83%
Average % Positive	42.33%	Average % Positive w/o ML5 & ML2-Slow				49.90%			
Average % False Alarm	57.67%	Average % False Alarm w/o ML5 & ML2-Slow				50.10%			

Start Track = 6	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
True Positive	11	21	25	14	2	13	13	11	12
False Positive	8	13	12	9	10	60	8	10	11
False Negative	1	0	0	2	2	3	0	0	2
True Negative	20	64	16	20	22	26	19	0	20
False Alarm	9	13	12	11	12	63	8	10	13
Total	20	34	37	25	14	76	21	21	25
% Positive	55.00%	61.76%	67.57%	56.00%	14.29%	17.11%	61.90%	52.38%	48.00%
% False Alarm	45.00%	38.24%	32.43%	44.00%	85.71%	82.89%	38.10%	47.62%	52.00%
% False Positive	40.00%	38.24%	32.43%	36.00%	71.43%	78.95%	38.10%	47.62%	44.00%
Average % Positive	48.22%	Average % Positive w/o ML5 & ML2-Slow				57.52%			
Average % False Alarm	51.78%	Average % False Alarm w/o ML5 & ML2-Slow				42.48%			

Start Track = 5	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	12	22	26	15	0	13	14	12	12
False Positive	9	13	12	9	13	61	8	10	11
False Negative	0	0	0	2	2	2	0	0	3
False Alarm	9	13	12	11	15	63	8	10	14
Total	21	35	38	26	15	76	22	22	26
% Positive	57.14%	62.86%	68.42%	57.69%	0.00%	17.11%	63.64%	54.55%	46.15%
% False Alarm	42.86%	37.14%	31.58%	42.31%	100.00%	82.89%	36.36%	45.45%	53.85%
% False Positive	42.86%	37.14%	31.58%	34.62%	86.67%	80.26%	36.36%	45.45%	42.31%
Average % Positive	47.51%	Average % Positive w/o ML5 & ML2-Slow				58.64%			
Average % False Alarm	52.49%	Average % False Alarm w/o ML5 & ML2-Slow				41.36%			

Start Track = 4	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	12	22	26	14	0	13	14	12	12
False Positive	9	13	12	9	13	61	8	10	11
False Negative	0	0	0	3	2	2	0	0	3
False Alarm	9	13	12	12	15	63	8	10	14
Total	21	35	38	26	15	76	22	22	26
% Positive	57.14%	62.86%	68.42%	53.85%	0.00%	17.11%	63.64%	54.55%	46.15%
% False Alarm	42.86%	37.14%	31.58%	46.15%	100.00%	82.89%	36.36%	45.45%	53.85%
% False Positive	42.86%	37.14%	31.58%	34.62%	86.67%	80.26%	36.36%	45.45%	42.31%
Average % Positive	47.08%	Average % Positive w/o ML5 & ML2-Slow				58.09%			
Average % False Alarm	52.92%	Average % False Alarm w/o ML5 & ML2-Slow				41.91%			

Start Track = 3	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	14	24	0	15	0	14	16	14	13
False Positive	9	13	37	9	16	62	8	10	11
False Negative	0	0	3	4	1	2	0	0	4
False Alarm	9	13	40	13	17	64	8	10	15
Total	23	37	40	28	17	78	24	24	28
% Positive	60.87%	64.86%	0.00%	53.57%	0.00%	17.95%	66.67%	58.33%	46.43%
% False Alarm	39.13%	35.14%	100.00%	46.43%	100.00%	82.05%	33.33%	41.67%	53.57%
% False Positive	39.13%	35.14%	92.50%	32.14%	94.12%	79.49%	33.33%	41.67%	39.29%
Average % Positive	40.96%	Average % Positive w/o ML5 & ML2-Slow				50.10%			
Average % False Alarm	59.04%	Average % False Alarm w/o ML5 & ML2-Slow				49.90%			

Skip Track = 2	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	15	0	0	15	3	15	17	14	13
False Positive	9	37	38	10	12	62	8	11	11
False Negative	0	1	3	4	3	2	0	0	5
False Alarm	9	38	41	14	15	64	8	11	16
Total	24	38	41	29	18	79	25	25	29
% Positive	62.50%	0.00%	0.00%	51.72%	16.67%	18.99%	68.00%	56.00%	44.83%
% False Alarm	37.50%	100.00%	100.00%	48.28%	83.33%	81.01%	32.00%	44.00%	55.17%
% False Positive	37.50%	97.37%	92.68%	34.48%	66.67%	78.48%	32.00%	44.00%	37.93%
Average % Positive	35.41%	Average % Positive w/o ML5 & ML2-Slow				40.44%			
Average % False Alarm	64.59%	Average % False Alarm w/o ML5 & ML2-Slow				59.56%			

Skip Track = 1	ML1	ML3	ML4	ML6	ML5	ML2-Slow	ML1 L-R	ML1 R-L	ML6 U-D
Positive	16	0	0	16	4	16	18	1	14
False Positive	9	38	41	10	12	62	8	25	12
False Negative	0	1	1	4	3	2	0	0	4
False Alarm	9	39	42	14	15	64	8	25	16
Total	25	39	42	30	19	80	26	26	30
% Positive	64.00%	0.00%	0.00%	53.33%	21.05%	20.00%	69.23%	3.85%	46.67%
% False Alarm	36.00%	100.00%	100.00%	46.67%	78.95%	80.00%	30.77%	96.15%	53.33%
% False Positive	36.00%	97.44%	97.62%	33.33%	63.16%	77.50%	30.77%	96.15%	40.00%
Average % Positive	30.90%	Average % Positive w/o ML5 & ML2-Slow				33.87%			
Average % False Alarm	69.10%	Average % False Alarm w/o ML5 & ML2-Slow				66.13%			

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